PLASTICITY OF ANISOTROPIC MEDIA

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Assume that in a rectangular Cartesian coordinate system x, y, z the stress—strain state of the medium is characterized by symmetrical tensors of rank 2, T_σ and T_ε, and the relations between the elastic deformations and stresses represent the generalized Hooke's law

$$T_\varepsilon = D \cdot\cdot T_\sigma, \tag{1}$$

where D is the tensor of elastic compliances of rank 4.

Let the tensors T_i (here, the indices i and j everywhere assume the values 1, ..., 6) form an orthonormal system of characteristic tensors of the tensor D, while $1/\lambda_i$ are the characteristic values [1, 2]:

$$D \cdot\cdot T_i = T_i/\lambda_i,\ (T_i,\ T_j) = \delta_{ij}. \tag{2}$$

We expand T_σ and T_ε with respect to the basis tensors T_i:

$$T_\varepsilon = E_i T_i,\ T_\sigma = S_i T_i. \tag{3}$$

It follows from (1)-(3) that

$$E_i = S_i/\lambda_i. \tag{4}$$

We introduce the following definition. We shall refer to the axes in tensor space, determined by the basis tensors T_i, as the axes of anisotropy of the starting material.

The anisotropy axes at the stage of elastic deformation of an element of the medium are determined by the values of the elastic compliances of the materials and do not depend on the values of the tensors T_σ and T_ε.

We shall make the following basic assumption: The orientation of the anisotropy axes in tensor space also remains unchanged with the appearance of plastic deformations.

In order to examine the variants of the theory of plastic flow, we must introduce into the analysis the tensors of increments of stresses, strains, and plastic deformations $T_{\Delta\sigma}$, $T_{\Delta\varepsilon}$, and $T_{\Delta\varepsilon_p}$, respectively. In the T_i basis, we have

$$T_{\Delta\sigma} = \Delta S_i T_i,\quad T_{\Delta\varepsilon} = \Delta E_i T_i,\quad T_{\Delta\varepsilon_p} = \Delta E_i^p T_i.$$

We shall investigate the possible cases.

1. Case of Total Anisotropy. Assume that all characteristic values are different

$$\lambda_1 \neq \lambda_2 \neq \lambda_3 \neq \lambda_4 \neq \lambda_5 \neq \lambda_6.$$

We shall examine the inequalities (4). If the anisotropic material deforms elastically, then the values of λ_i are constant and the dependences $S_i = S_i(E_i)$ are linear (see Fig. 1). When

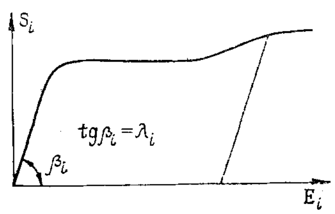

Fig. 1

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some of the values of S_i attain and exceed some values $S^o{}_i$,

$$|S_i| \geqslant S_i^0$$

($S^o{}_i$ — the finite or infinite yield point of the anisotropic material along the axis with unit vector T_i is established experimentally) plastic changes in the deformation E_i will occur and the dependences $S_i = S_i(E_i)$ will no longer be linear. The general form of the strain diagram $S_i = S_i(E_i)$ is shown in Fig. 1. When the load is removed, we will assume, as usual, that the unloading follows an elastic law.

The equations of the classical theories for one-dimensional situations are well known. They take into account both the isotropic and anisotropic nature of the hardening along the corresponding strain axes. We shall restrict ourselves here to the case of isotropic hardening, for which the basic equations of these theories have the following form:

Deformation theory of plasticity

$$E_i = S_i/\lambda_i^c, \tag{5}$$

where $\lambda^c{}_i = \lambda^c{}_i(S_i)$ is the intersecting modulus on the diagram $S_i = S_i(E_i)$;

Theory of plastic flow

$$\Delta E_i^p = \Delta S_i/\lambda_i^p, \tag{6}$$

where $\lambda^p{}_i = \lambda^p{}_i(S_i)$ is the tangential modulus on the diagram $S_i = S_i(E_i^p)$;

Theory of ideal plasticity

$$|S_i| = S_i^0. \tag{7}$$

Equations (5)-(7) are valid under the conditions that $S_i \Delta S_i \geqslant 0$ (summation is not implied!); if $S_i \Delta S_i \leq 0$, then elastic unloading will occur along the corresponding axes according to the law $\Delta E_i = \Delta S_i/\lambda_i$.

2. Case of Partial Isotropy. Assume that some of the characteristic values are equal:

$$\lambda_k = \lambda_m = \dots = \lambda_{km} \dots (k \neq m \neq \dots).$$

In this case, from equalities (3) and (4) we find

$$E_k T_k + E_m T_m + \dots = (S_k T_k + S_m T_m + \dots)/\lambda_{km} \dots \tag{8}$$

We shall call the subspace defined by the basis tensors T_k, T_m, ..., the isotropic subspace.

In the case of the usual Hooke's law ($\lambda_1 = \dots = \lambda_5 \neq \lambda_6$) the isotropic subspace coincides with the deviator subspace.

The construction of the equations of the classical theories in isotropic subspaces does not involve any difficulties. For clarity, we shall examine, for example, the case when $\lambda_k = \lambda_m = \lambda_{km}$. In this case the isotropic subspace is a plane of isotropy.

We introduce the following tensors:

$$t_\varepsilon = E_k T_k + E_m T_m, \quad t_\sigma = S_k T_k + S_m T_m.$$

It follows from (8) that

$$t_\varepsilon = t_\sigma/\lambda_{km}. \tag{9}$$

We now introduce the polar coordinates of the tensors t_ε and t_σ:

$$E_{km} = \sqrt{E_k^2 + E_m^2}, \quad \operatorname{tg} 2\Omega_{km} = E_m/E_k,$$

$$S_{km} = \sqrt{S_k^2 + S_m^2}, \quad \operatorname{tg} 2\theta_{km} = S_m/S_k.$$

It follows from the equality (9) that

$$E_{km} = S_{km}/\lambda_{km}, \quad \Omega_{km} = \theta_{km}.$$

If the anisotropic material is deformed elastically in the isotropy plane under examination (there can be several isotropy planes), then λ_{km} is a constant and the dependence $S_{km} = S_{km}(E_{km})$ is linear. When S_{km} attains and exceeds the yield stress,

$$S_{km} \geqslant S^0_{km},$$

the dependence is no longer linear. The general form of the strain $S_{km} = S_{km}(E_{km})$ is the same as in the figure. When the load is removed, we assume that the unloading follows an elastic law.

We write down the equations of classical theories in the case under examination, assuming that the hardening has an isotropic character:

Deformation theory of plasticity

$$t_\varepsilon = t_\sigma / \lambda^c_{km}, \tag{10}$$

where $\lambda^c_{km} = \lambda^c_{km}(S_{km})$ is the intersecting modulus on the diagram $S_{km} = S_{km}(E_{km})^0$, obtained with a proportional load in the plane of isotropy; the tensor equality (10) is equivalent to two scalar equalities:

$$E_{km} = S_{km}/\lambda^c_{km}, \quad \Omega_{km} = \theta_{km};$$

Theory of plastic flow

$$t_{\Delta\varepsilon_p} = t_\sigma \frac{(t_{\Delta\sigma}, t_\sigma)}{(t_\sigma, t_\sigma)} \bigg/ \lambda^p_{km}, \tag{11}$$

where $\lambda^p_{km} = \lambda^p_{km}(S_{km})$ is the tangential modulus on the curve $S_{km} = S_{km}(E_{km})^0$, obtained with a proportional load in the isotropy plane;

$$t_{\Delta\varepsilon_p} = \Delta E^p_k T_k + \Delta E^p_m T_m; \quad t_{\Delta\sigma} = \Delta S_k T_k + \Delta S_m T_m;$$

Theory of ideal plasticity

$$S_{km} = S^0_{km}, \quad \Delta E^p_k / S_k = \Delta E^p_m / S_m. \tag{12}$$

Equations (10)-(12) are valid under the condition that $(t_{\Delta\sigma}, t_\sigma) \geq 0$; if $(t_{\Delta\sigma}, t_\sigma) \leq 0$, then the elastic unloading obeys the law $t_{\Delta\varepsilon} = t_{\Delta\sigma}/\lambda_{km}$.

Various other models of complex loading can be examined in the isotropy planes analogously to [3-6]; the condition $S_{km} = S^o_{km}$ is approximated by piecewise linear surfaces.

We note that if the set of characteristic values consists of simple and multiple roots, then the system of equations describing elastoplastic deformation of the anisotropic medium is a combination of systems of equations of two types: systems of equations describing the process of deformation along corresponding anisotropy axes (the construction is performed as in case 1), and systems of equations describing the deformation in corresponding isotropic subspaces.

3. Case of Complete Isotropy. Assume that all roots are equal:

$$\lambda_1 = \lambda_2 \ldots = \lambda_6.$$

Case 3 is obviously a particular case of case 2, and the construction of the equations of the classical theory of plasticity here is entirely analogous to the preceding case. In case 3 the entire space examined will be isotropic.

We note in conclusion that most elastoplastic models are written in the form

$$T_{\Delta\varepsilon} = D \cdot\cdot T_{\Delta\sigma},$$

so that the analysis presented above can be used to reject some of the existing models and to construct new ones.

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FLEXURAL—GRAVITATIONAL WAVES FROM MOVING DISTURBANCES

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We investigate propagating flexural—gravitational waves, generated under the action of a load moving over the surface of a floating elastic plate, found in a state of uniform extension or compression. Without account of extension or compression stresses, flexural—gravitational propagating waves were considered in [1, 2]. Planar waves were investigated in [3, 4] under conditions of longitudinal compression.

1. Let a thin, isotropic, elastic plate float on the surface of an ideal, incompressible liquid of finite depth H. The plate and the liquid are not restricted in their horizontal stresses. The plate is displaced across the surface with a velocity v of the loading $p = p_0 f(x_1, y)$, $x_1 = x + vt$. Consider the effect of a uniform extension on the generated flexural—gravitational marine wave, assuming that the liquid motion is a potential flow, and that the velocities of the liquid particle motion and of the plate deflection ζ are low.

Taking into account uniform extension [5-7] in a coordinate system x_1, y, related to the moving pressure region, the problem reduces to solving the Laplace equation for the velocity potential φ

$$\Delta\varphi = 0,\ -H < z < 0,\ -\infty < x,\ y < \infty \tag{1.1}$$

with boundary conditions

$$D_1\nabla^4\zeta - Q_1\Delta_l\zeta + \varkappa_1 v^2\frac{\partial^2\zeta}{\partial x^2} + \zeta + \frac{v}{g}\frac{\partial\varphi}{\partial x} = -p_1 f(x, y) \quad \text{at} \quad z = 0, \tag{1.2}$$
$$\partial\varphi/\partial z = 0 \quad \text{at} \quad z = -H,$$

where

$$D_1 = D/\rho g,\ Q_1 = Q/\rho g,\ \varkappa_1 = \rho_1 h/\rho g,\ D = Eh^3/[12(1-\mu^2)],\ p_1 = p_0/\rho g,$$
$$\nabla^4 = \Delta_l^2,\quad \Delta_l = \partial^2/\partial x^2 + \partial^2/\partial y^2,$$

ρ, liquid density; E, h, ρ_1, and μ, respectively, the normal elastic modulus, the width, density, and Poisson coefficient of the plate; Q, extension stress; ζ and φ, related by the mathematical condition $\varphi_z = v\zeta_x$ at z = 0. From here on the subscript 1 of x_1 will be omitted.

Applying a Fourier transform in horizontal coordinates to solve the problem (1.1), (1.2), we obtain, in the case of an axisymmetric load, an integral representation for the plate deflection (raising the plate-liquid surface):

$$\zeta = \frac{1}{2\pi} p_1 \operatorname{Re}\left\{\int_0^\infty \frac{r}{\tau}\bar{f}(r) M(r) J(r, R, \gamma)\, dr\right\}; \tag{1.3}$$

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